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ASYMPTOTIC ANALYSIS AND NUMERICAL SOLUTION OF THE TWO-LEVEL BOUNDARY EQUATIONS OF A PLANE PROBLEM OF STATIONARY HYDROELASTICITY†

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For the two-level version of the method of boundary integral equations applied to the analysis of oscillations of composite thin-shelled constructions in an acoustic medium [1–4] the asymptotic analysis and simplification of equations in several characteristic excitation bands is carried out within the framework of the plane problem. The results are compared with those of a numerical solution of the problem.

UNDER the cylindrical bending conditions, we consider the oscillations of a plate subject to a load $q(x, t)$ not connected with the presence of an acoustic medium and being the source of vibrations. The time dependence of the external load q and all the other functions to be introduced are taken to be of the form $e^{-i\omega t}$ the time multiplier being omitted.

The boundary equation for the acoustic contact pressure can be written as follows [5, 6]:

$$p(x)=\frac{i}{2}\left(\frac{\omega l}{c}\right)^2\frac{\rho_0}{\rho}\int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)w(\xi)\,d\xi \tag{1}$$

The equation describing the oscillations of the plate has the form [5, 9]

$$w^{\mathrm{IV}}(x)-\mu^4 w(x)=12(1-\nu^2)(l/h)^3\,[q(x)+p(x)] \tag{2}$$

$$\mu^4=12(1-\nu^2)\rho\,\omega^2 l^4/(Eh^2)$$

Here h is the thickness of the plate, ν is Poisson's ratio, E is the longitudinal elasticity modulus of the material of the plate, ρ is the density of the plate, and l is the length of the plate (a prime designates the derivative with respect to x).

The boundary conditions (two conditions at each of the points $x=0$ and $x=l$) can be arbitrary.

When using the boundary equation method, in addition to the state of the plate to be determined, which is described by Eq. (2) with boundary conditions, one must introduce an auxiliary state. The choice of the auxiliary state is, to a large extent, arbitrary and is dictated by convenience and the compactness of the solution. The simplest choice is to use Green's function corresponding to a concentrated force applied to an infinite construction [2]. In the

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case under consideration, such a function must be a solution of the equation (in the dimensionless form)

$$\partial^4 W(x, \xi)/\partial \xi^4 - \mu^4 W(x, \xi) = \delta(x-\xi) \tag{3}$$

In the boundary equation method solutions corresponding to the absence of any sources at infinity (satisfying the radiation principle) are usually used as the fundamental solutions. In the case in hand, in which the solution for an infinite domain is used just as the fundamental solution to construct the solution for a plate of finite dimensions, the radiation principle does not have to be fulfilled. Indeed, one has only to verify the inhomogeneous equations in the domain with the aid of Green's function for an infinite plate. The boundary conditions can be met by means of additional homogeneous solutions defined by the boundary conditions, the total solution being unique and independent of the choice of the first part (i.e. the inhomogeneous solution in the domain). This makes it possible to use the real function

$$W(x, \xi) = -\tfrac{1}{4}\mu^{-3}\,[\sin(\mu\,|x-\xi|) + \exp(-\mu\,|x-\xi|)] \tag{4}$$

which satisfies the inhomogeneous equation (3), as the fundamental solution. For the problem on the oscillations of a plate subject to the cylindrical bending conditions, the boundary equations follow directly from Betti's theorem in [5]

$$\int_0^1 [M^0(x, \xi)\, w''(\xi) + \rho h \omega^2 W(x, \xi)\, w(\xi)]\, d\xi =$$

$$= \int_0^1 \left[M(\xi) \frac{\partial^2 W(x, \xi)}{\partial \xi^2} + \rho h \omega^2 w(\xi) W(x, \xi)\right] d\xi \tag{5}$$

Here D is the cylindrical stiffness, the functions M and w correspond to the state of the plate of finite dimensions to be determined, and M^0 and W correspond to the auxiliary state of the plate considered as an infinite one.

On integrating (5) by parts, and using formulae (3) and (4), we obtain the following integral representation (resembling the Somigliana formula in elasticity theory)

$$w(x) = 12(1-\nu^2)\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi) +$$

$$+ p(\xi, 0)]\; W(x, \xi)\, d\xi + [w''(\xi) \frac{\partial W(x, \xi)}{\partial \xi}$$

$$- w'''(\xi)\, W(x, \xi) - w'(\xi) \frac{\partial^2 W(x, \xi)}{\partial \xi^2} +$$

$$+ w(\xi) \frac{\partial^3 W(x, \xi)}{\partial \xi^3}]\Big|_{\xi=0}^{\xi=1} \tag{6}$$

Formula (6) is insufficient to construct the solution of the equation describing the oscillations of the plate. One needs an integral representation for the rotation angle [5]. The latter is easily obtained from (6) by differentiating with respect to the coordinate of the observation point x

$$w'(x) = 12(1-\nu^2)\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi) +$$

$$+p(\xi,0)]\ \frac{\partial W(x,\xi)}{\partial x}\ d\xi+[w''(\xi)\ \frac{\partial^2 W(x,\xi)}{\partial x\,\partial\xi}-$$

$$-w'''(\xi)\ \frac{\partial W(x,\xi)}{\partial x}\ -\ w'(\xi)\ \frac{\partial^3 W(x,\xi)}{\partial x\,\partial\xi^2}\ +$$

$$+w(\xi)\ \frac{\partial^4 W(x,\xi)}{\partial x\,\partial\xi^3}\Big|\Big|_{\xi=0}^{\xi=1} \tag{7}$$

Now, letting the observation point x tend to $\xi=0$ and $\xi=1$ in (6) and (7), we obtain four boundary equations with eight algebraic unknowns, namely, $w(\xi), w'(\xi), w''(\xi), w'''(\xi)$; $\xi=0; 1$; and the unknown contact pressure $p(\xi,0)$

$$\frac{1}{2}w(0)-\frac{1}{4}\alpha_1 w(1)+\frac{1}{4\mu}\ w'(0)+\frac{1}{4\mu}\ w'(1)+$$

$$+\frac{1}{4\mu^3}\ w'''(0)-\frac{1}{4\mu^3}\ \alpha_3 w'''(1)+\frac{1}{4\mu^2}\ \alpha_4 w''(1)=$$

$$=-\ \frac{3(1-\nu^2)}{\mu^3}\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi)+p(\xi,0)]\ [\sin\mu\xi+\exp(-\mu\xi)]\ d\xi$$

$$-\frac{1}{4}\alpha_1\ w(0)+\frac{1}{2}\ w(1)-\frac{1}{4\mu}\ \alpha_2\ w'(0)-\frac{1}{4\mu}\ w'(1)+$$

$$+\frac{1}{4\mu^3}\ \alpha_3\ w'''(0)-\frac{1}{4\mu^3}\ w'''(1)+\frac{1}{4\mu^2}\ \alpha_4 w''(0)=$$

$$=-\ \frac{3(1-\nu^2)}{\mu^3}\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi)+p(\xi,0)]\ [\sin\mu(1-\xi)+\exp(-\mu(1-\xi))]\ d\xi$$

$$\frac{1}{2}\ w'(0)-\frac{1}{4}\alpha_1 w'(1)+\frac{\mu}{4}\ w(0)-$$

$$-\frac{\mu}{4}\ \alpha_3 w(1)+\frac{1}{4\mu^2}\ \alpha_4\ w'''(1)+\frac{1}{4\mu}\ w''(0)+\frac{1}{4\mu}\ \alpha_2 w''(1)=$$

$$=-\ \frac{3(1-\nu^2)}{\mu^2}\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi)+p(\xi,0)]\ [\cos\mu\xi-\exp(-\mu\xi)]\ d\xi$$

$$-\frac{1}{4}\ \alpha_1 w'(0)+\frac{1}{2}\ w'(1)+\frac{\mu}{4}\ \alpha_3 w(0)-$$

$$-\frac{\mu}{4}\ w(1)\ +\frac{1}{4\mu^2}\ \alpha_4 w'''(0)-\frac{1}{4\mu}\ \alpha_2 w''(0)-$$

$$-\ \frac{1}{4\mu}\ w''(1)=-\frac{3(1-\nu^2)}{\mu^2}\left(\frac{l}{h}\right)^3 \int_0^1 [q(\xi)+$$

$$+p(\xi,0)]\ [\cos\mu(1-\xi)-\exp(-\mu(1-\xi))]\ d\xi$$

$$\alpha_1 = \beta_1 + \beta_3, \quad \alpha_2 = \beta_2 - \beta_3, \quad \alpha_3 = \beta_2 + \beta_3, \quad \alpha_4 = \beta_1 - \beta_3 \tag{8}$$
$$\beta_1 = \cos\mu, \quad \beta_2 = \sin\mu, \quad \beta_3 = \exp(-\mu)$$

The system of boundary equations (8) can be closed by the four boundary conditions and the integral equation for the contact pressure, which can be obtained by substituting (6) into (1)

$$p(x,0) = \frac{i}{2}\left(\frac{\omega l}{c}\right)^2 \frac{\rho_0}{\rho} \left\{ 12(1-\nu^2)\left(\frac{l}{h}\right)^3 \int_0^1\int_0^1 \times \right.$$

$$\times H_0^{(1)}\left(\frac{\omega l}{c_0} |x-\xi|\right) W(\xi,u)\,[q(u)+p(u,0)]\,d\xi\,du +$$

$$+ [w''(u) \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0} |x-\xi|\right) \frac{\partial W(\xi,u)}{\partial u}\,d\xi - w'''(u) \times$$

$$\times \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0} |x-\xi|\right) W(\xi,u)\,d\xi - w'(u) \times$$

$$\times \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0} |x-\xi|\right) \frac{\partial^2 W(\xi,u)}{\partial u^2}\,d\xi + w(u) \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0} |x-\xi|\right) \times$$

$$\left. \times \frac{\partial^3 W(\xi,u)}{\partial u^3}\,d\xi] \,\Big|_{u=0}^{u=1} \right\} \tag{9}$$

The relations (8), (9) form a system of resolving equations of the two-level method of boundary integral equations applied to the plane problem on the oscillations of a plate of finite length placed on a rigid screen and being in contact with an acoustic medium. The simplest way of solving the problem consists of a piecewise constant approximation of the function $p(x, 0)$ to be determined and a numerical inversion of a system of linear algebraic equations of order $N+8$, where N is the number of segments along the plate, the amplitudes p_n, $n=1, 2, \ldots, N$ being assumed constant within each of the segments.

The boundary equations of the first level (Eqs (8) for the oscillations of the plate) are algebraic equations with respect to the generalized boundary forces and displacements as well as with respect to the integrals of the given external load $q(\xi)$ and the reactive acoustic load $p(\xi, 0)$.

Moreover, for each specific way of fixing the ends of the plate, only four boundary conditions and (or) displacements remain unknown. This makes it possible to solve the system of first-level equations analytically and to reduce the problem to the single integral equation (9) with the bending parameters on the edges of the plate excluded. Obtaining such an equation in the exact analytic form makes it possible to carry out a detailed asymptotic analysis of the interaction between the plate of finite length and the acoustic medium in any excitation band, including the principal resonance frequencies (the resonance frequencies of oscillations of the plate in vacuum).

As an example, we consider a freely supported plate. Because the plate is fixed in a symmetric way, its motion under any longitudinal load is the sum of the symmetric and skew-symmetric components relative to the centre of the plate. Then the system of equations (8) splits into two corresponding second-order systems.

Substituting the solution of the equations for the symmetric oscillations relative to the centre of the plate into (9), we get

$$p(x,0) + \frac{i}{8}\frac{\rho_0}{\rho}\mu\frac{l}{h}\left\{\int_0^1\int_0^1 [q(u)+p(u,0)]\, H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right) \times\right.$$

$$\times [\sin\mu|u-\xi| + \exp(-\mu|u-\xi|)]\, du\, d\xi - \frac{1}{\sin\mu}\int_0^1 [q(u)+$$

$$+p(u,0)]\sin\mu u\, du \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)[\sin\mu\xi + \sin\mu(1-\xi)]\, d\xi -$$

$$-\frac{1}{1+\exp(-\mu)}\int_0^1 [q(u)+p(u,0)]\exp(-\mu u)\, du \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right) \times$$

$$\left.\times [\exp(-\mu\xi) + \exp(-\mu(1-\xi))]\, d\xi\right\} = 0 \tag{10}$$

Equation (10) defines the acoustic pressure on the surface of a freely supported plate subject to an arbitrary symmetric load. Since the equation describing the skew-symmetric oscillations of the plate has the same structure as (10), we shall restrict ourselves to the asymptotic analysis of the latter.

If the connection between the vibration amplitudes of the plate and the acoustic pressure on the surface of the plate is neglected ($p(u, 0)$ is omitted in (10)), then the equation turns into the simple computational formula

$$p(x,0) = -\frac{i}{8}\frac{\rho_0}{\rho}\mu\frac{l}{n}\left\{\int_0^1\int_0^1 q(u) H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right) \times\right.$$

$$\times [\sin\mu|u-\xi| + \exp(-\mu|u-\xi|)]\, du\, d\xi - \frac{1}{\sin\mu}\int_0^1 q(u)\sin\mu u\, du \times$$

$$\times \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)[\sin\mu\xi + \sin\mu(1-\xi)]\, d\xi -$$

$$-\frac{1}{1+\exp(-\mu)}\int_0^1 q(u)\exp(-\mu u)\, du \times$$

$$\left.\times \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)[\exp(-\mu\xi) + \exp(-\mu(1-\xi))]\, d\xi\right\} \tag{11}$$

The first integral defines the pressure produced during the oscillations of the considered portion of an unbounded plate, while the second and third integrals correspond to boundary effect, namely, the contribution of the rotation angles at the ends of the plate and the reaction of the support into the total pressure.

If the constraints of the problem are taken into account, then (in the case of a piecewise-constant approximation of the contact pressure) the matrix of the system of algebraic equations with respect to the amplitudes p_n $(n = 1, 2, \ldots, N)$ ceases to be diagonal.

When (10) is used, each of the off-diagonal terms appearing in this matrix is also the sum of two components: the component corresponding to the interaction of the considered portion of the unbounded plate with the acoustic medium and the boundary effect integrals

$$I_1 = \frac{i}{8}\frac{\rho_0}{\rho}\mu\frac{l}{h}\int_0^1\int_0^1 p(u,0) H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right) \times$$

$$\times\ [\sin\mu\,|u-\xi| + \exp(-\mu\,|u-\xi|)]\ du d\xi \tag{12}$$

$$I_2 = -\frac{i}{8}\frac{\rho_0}{\rho}\mu\frac{l}{h}\left\{\frac{1}{\sin\mu}\int_0^1 p(u,0)\sin\mu u du\int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)\times\right.$$

$$\times\ [\sin\mu\xi + \sin\mu(1-\xi)]\ d\xi + \frac{1}{1+\exp(-\mu)}\int_0^1 p(u,0)\times$$

$$\times \exp(-\mu u) du \int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)\times$$

$$\left.\times\ [\exp(-\mu\xi) + \exp(-\mu(1-\xi))]\ d\xi\right\} \tag{13}$$

These components are proportional to the coefficient $\kappa = \frac{1}{8}(\rho_0/\rho)(l/h)\mu$. Taking into account that $\nu = 0.3$, and $\rho_0/\rho = 0.128$ for the pair consisting of steel and water, we have $\kappa \sim 0.029\,(l/h)^{3/2}$. Thus, for $l/h = 100$ we have $\kappa = 29\sqrt{(\omega l/c)}$. Even though the dependence of (12) and (13) on the characteristic parameters l/h and $\omega l/c$ is more complicated (the arguments of the integrands also contain these quantities), such an estimate substantiates the claim that in many cases the computation of the contact pressure from the simple formula (11) can lead to large errors.

In the integrals (12) and (13) the components corresponding to the homogeneous and inhomogeneous waves present in an infinite plate are specified explicitly, which makes it possible to estimate their contribution in each case.

As the characteristic frequencies providing a scale for dividing the whole excitation band into the basic segments we choose the first resonance frequency ω_r of oscillations of an isolated (dry) plate, at which $\mu_r^4 = \pi^4$ and the so-called compatibility frequency ω_g [7], at which the length of the plate is equal to the wavelength in the acoustic medium

$$\frac{\omega_r l}{c} = \frac{\pi^2}{12(1-\nu^2)}\left(\frac{h}{l}\right)^2; \qquad \frac{\omega_g l}{c} = 12(1-\nu^2)\left(\frac{c_0}{c}\right)^2\left(\frac{l}{h}\right)$$

One can therefore distinguish four basic frequency bands

1. extremely low frequencies

$$\omega l/c < (h/l)^2, \qquad \mu < [12(1-\nu^2)]^{1/4}(h/l)^{1/2}$$

2. low frequencies

$$\omega l/c < h/l, \qquad \mu < 1$$

3. intermediate frequencies

$$\omega l/c \sim 1, \qquad \mu \sim (l/h)^{1/2}$$

4. high frequencies

$$\omega l/c > (l/h), \qquad \mu > l/h$$

In the extremely low frequency band the Hankel function can be replaced by its two-term asymptotic forms

$$H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right) = 1 + \frac{2i\gamma}{\pi} +$$

$$+\frac{2i}{\pi}\ln\frac{\omega l}{c_0}+\frac{2i}{\pi}\ln|x-\xi| \tag{14}$$

and the functions representing the fundamental solution of the equation describing the oscillations of the plate have the form

$$\sin\mu x=\mu x, \qquad \exp(-\mu x)=1-\mu x \tag{15}$$

The substitution of (14) and (15) into (11) leads to the exact equality $p(x, 0)=0$. This means that in the band under consideration the acoustic medium does not interact with the plate as the plate oscillates: the load acting on the plate is quasistatic.

In the low-frequency band the representation (14) can be used as before, and one has to keep the quadratic terms of the expansion (15) when describing the oscillations of the plate.

If Eq. (11) is integrated over the length of the plate, then one can obtain an algebraic equation connecting the integrals

$$\int_0^1 p(u, 0)\,du \text{ and } \int_0^1 p(u, 0)\,u^2du$$

with the analogous integrals of the external load. Another equation of the same form can be obtained in the same way on integrating (11) supplemented with x^2.

It follows that the problem can be reduced to solving a system of second-order linear equations, the coefficients of which are computed explicitly. The reduction is quite cumbersome and will not be presented here. The substitution of the solution of this system into (10) turns the latter into an asymptotically exact expression for the acoustic contact pressure on the surface of the plate in terms of the external load. In the frequency band in question the pressure in the liquid is determined by the integral characteristics of the liquid, namely, the resultant force and the second central moment, rather than by the specific way of applying the external load.

The formula can prove to be important in practice, since the frequency band under consideration is close to the frequency corresponding to the maximum of the imaginary part of the displacement amplitude and, consequently, the maximum of the radiation power, i.e. the resonance frequency of oscillations in the liquid.

As a result of retaining the subsequent terms in the expansion of the fundamental solution $W(x, \xi)$ when higher frequencies are considered, the convolution

$$\int_0^1 H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)W(|\xi-u|)\,d\xi$$

turns out to be a transcendental function not only of x and u, but also of the distance $|x-u|$. This makes it impossible to represent the convolution as the sum of products $f_1(x)f_2(x)$ and to avoid solving the integral equation.

The asymptotic representations of the fundamental solutions cannot be used in the intermediate frequency band. The piecewise approximation of the contact pressure followed by the numerical solution of Eq. (10) turns out to be the only way of solving the problem. This frequency band may contain (for sufficiently short plates) the first dry resonance. The computation using (10) is impossible for $\mu=\pi$ because the denominator of the second integral term becomes equal to zero. However, the solution of the complete system (8), (9) does not present any difficulties at this frequency.

In the high-frequency band it is possible to use the following asymptotic forms of the Hankel function [8]

$$H_0^{(1)}\left(\frac{\omega l}{c_0}|x-\xi|\right)=\frac{\sqrt{2/\pi}}{\sqrt{(\omega l/c_0)}\sqrt{|x-\xi|}}\times$$

$$\times \exp\left[\frac{\omega l}{c_0}|x-\xi| - \frac{\pi}{4}\right] i \tag{16}$$

The convolution of (16) with those terms in (12) and (13) that represent homogeneous waves can be expressed in terms of Fresnel integrals. The inhomogeneous wave can be approximated by using the fact that $\mu \gg 1$ and

$$\exp(-\mu|\xi-u|) = 1-\mu|\xi-u| \tag{17}$$

in the frequency band under consideration. In doing so one must restrict the domain of integration in (12), (13) to the region $\mu|\xi-u|<1$, in which the two-term representation (17) is positive. This approach makes it possible to represent the contribution of the inhomogeneous wave in terms of Fresnel integrals, similarly as for the homogeneous wave.

Computations indicated that this simplified model is effective only for sufficiently high frequencies, at which is disputable whether or not the displacements and rotary inertia of the plate can be neglected. Since in the present paper the fundamental solution (4) is used for the plate within the framework of the Kirchhoff theory, no similar analysis of the oscillations in the liquid has been carried out in the high-frequency excitation band.

Figures 1(a–c) show the graphs of the oscillation amplitude of the plate in the middle of the span $w(l/2)$ as a function of the frequency parameter μ for $h/l = 0.001$; 0.01; 0.1 under uniformly distributed load. In all the graphs curve 1 corresponds to oscillations in vacuum, and curves 2 and 3 represent the real and imaginary parts of the oscillation amplitude in the liquid. Computations carried out in the low-frequency band reveal the passage through the first resonance in the liquid. This resonance is fixed by the change in the sign of the real part and the maximum of the imaginary part of the amplitude of oscillations, and also by the maximum of radiation power. Increasing the relative thickness of the plate results in extending the frequency band in which the magnitude of the imaginary part of the oscillation amplitude (and consequently also the radiation into the liquid) is essential. At the same time, the real part of the amplitude increases when the resonance is crossed. Moreover, increasing the thickness results in extending the frequency band in which formula (11) for the traditional acoustic computation of the contact pressure can be used: the inverse relation between the oscillation amplitudes of the plate and the contact pressure on the surface of the plate not important.

Curve 4 in Fig. 1(b) corresponds to the solution for low-frequency asymptotic forms. The error of such a computation remains small even in those cases when the frequency is much higher than the upper bound of the band for which the asymptotic solution is constructed.

There are no principal differences between the amplitude–frequency characteristics shown in Fig. 1 and those corresponding to the oscillations generated by a force concentrated in the middle of the span.

The values of the frequency parameter $(\omega l/c)\times 10^3$ corresponding to three resonances in the liquid obtained with the use of the proposed version of the boundary equation method are 21.6 (19.4 [4], 21.7 [9]), 202 (192 [4], 204 [9]), 527 (498 [4], 527 [9]). The values in parentheses have been obtained by various numerical methods [4, 9].

The analysis of the structure of the system of equations obtained from (11) as a result of a piecewise-constant approximation of the contact pressure makes it possible to draw the following conclusions.

In the low-frequency band neglect of the influence of the edges of the plate leads to substantial errors in determining the acoustic contact pressure: if the corrections (12) and (13) are taken into account simultaneously, the off-diagonal terms of the matrix are of the order of 10^{-3}, while the separate contributions of either of the corrections are of the order of 10^{-1}.

At frequencies close to the resonance excitation the effect of the acoustic medium becomes so large that the corrections (12) and (13) exceed the magnitude of the diagonal terms. In this case, both corrections are equally important, and neither of them can be neglected.

At intermediate frequencies (above the first dry resonance) the influence of the medium is still important, but, in principle, it is determined by the correction (12) and the integrals of the boundary effect (13) can be omitted. This argument remains valid for higher frequencies too.

Since the above arguments are based on the analysis of the left-hand side of equation (11), they are

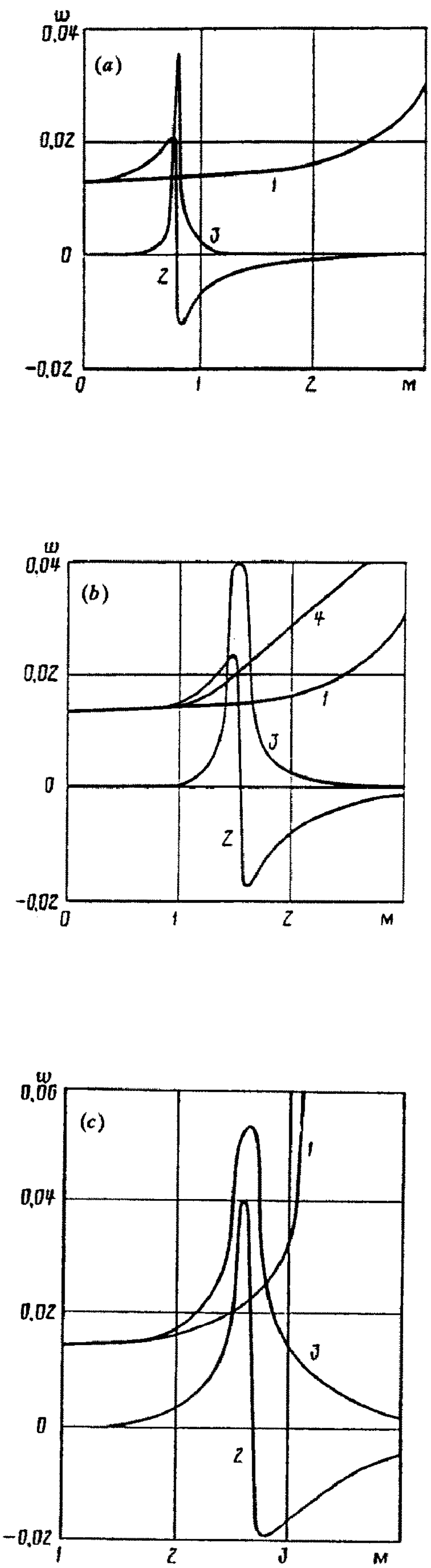

FIG. 1.

independent of the load applied to the plate.

The change in the distribution of pressure on the surface of the plate being in contact with an acoustic medium has been analysed in the two most characteristic cases of excitation of oscillations, namely, a concentrated force applied in the middle of the span and a uniformly distributed load. In these cases, two versions of the computation have been used: the acoustic approximation using formula (11) and the hydroelastic solution corresponding to the precise formulation of the contact problem.

Figure 2 shows the distribution of the real and imaginary parts A and B (curves 1 and 2) of contact pressure amplitudes under a concentrated unit force ($\mu = 0.266$). These curves have the same form both in the acoustic and hydroelastic setting, which can be explained by the lack of a connection with the problem of determining the vibration amplitudes. Moreover, the concentrated force spreads uniformly enough along the plate.

Figure 3 shows the pressure distribution under the action of a concentrated force at frequency $\mu = 1.456$, which is close to the first resonance in the liquid. For $\mu = 4.729$ similar curves are constructed in Fig. 4. The curves in Fig. 5 apply to oscillations generated by a uniformly distributed load of unit intensity at the same frequency $\mu = 4.729$. In Figs 3–5 the curves $A(1)$ correspond to the real part of the pressure in the acoustic approximation, while the curves $B(2)$ correspond to the real part of the pressure in the hydroelastic formulation of the problem. The imaginary parts are represented by the curves $C(3)$ and $D(4)$, respectively.

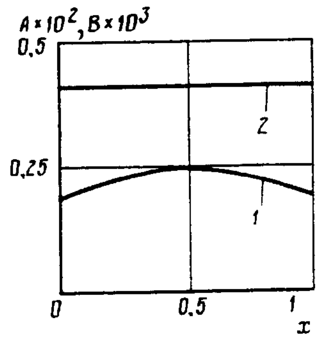

FIG. 2.

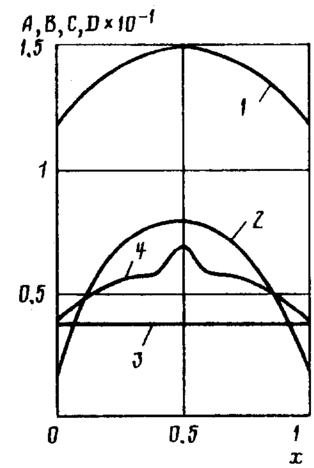

FIG. 3.

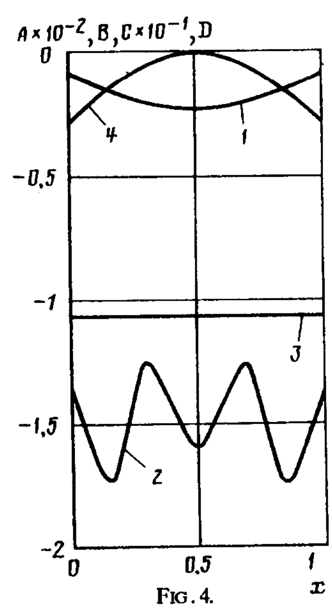

FIG. 4.

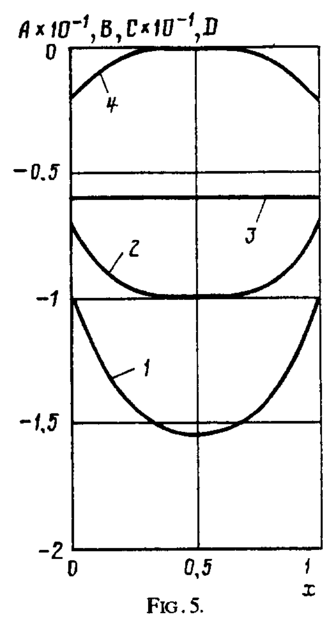

FIG. 5.

It can be seen that the acoustic approximation is inadmissible, because it leads to very large errors in determining the contact pressure, which is distributed over the surface of the plate in a rather complex way, especially in the case of intermediate frequency excitation ($\mu = 4.729$). The real part turns out to be oscillatory, while the imaginary part is localized near the edges of the plate (see Fig. 4). Under a uniform external load, the distribution of the contact pressure is more uniform. What is remarkable is that the external load is almost completely compensated in the middle part of the plate (curve 2) and the imaginary part of the pressure is localized in the neighbourhood of the edges of the plate (curve 4).

REFERENCES

1. SLEPYAN L. I. and SOROKIN S. V., The boundary integral equation method in hydroelasticity. *Izv. Akad. Nauk SSSR, MTT* 4, 166–176, 1989.
2. SLEPYAN L. I. and SOROKIN S. V., The system of boundary integral equations of the dynamics of composite thin-shelled constructions interacting with a liquid. *Izv. Akad. Nauk SSSR, MTT* 4, 113–121, 1990.
3. SOROKIN S. V., Oscillations of coaxial cylindrical shells of finite length in an acoustic medium. In *Applied Problems Involving Strength and Plasticity*, pp. 79–84. Gor'kii Gosp. Univ., Gor'kii, 1990.
4. SOROKIN S. V., Application of orthogonal sweep methods and boundary integral equations in the computation of hydroelastic oscillations of plates. In *Some Problems in Mechanics of Ship Construction*, pp. 67–73, Korablestroit. Inst., Leningrad, 1988.
5. BANERJEE P. and BUTTERFIELD R., *The Boundary Element Method in Applied Sciences*. Mir, Moscow, 1984.
6. SHENDEROV E. L., *Radiation and Scattering of Sound*. Sudostroyeniye, Leningrad, 1989.
7. CRIGHTON D. G. and INNES D., Structures under heavy fluid loading. *Proc. Roy. Soc. London, Ser. A* **312**, 1520–1521, 295–341, 1984.
8. ABRAMOWITZ M. and STEGUN I. M., *Handbook of Mathematical Functions*. Dover, New York, 1965.
9. GOLOVANOV V. A., POPOV A. L. and CHERNYSHEV G. I., Oscillations of a bounded plate in a liquid. *Prikl. Mat. Mekh.* **50**, 436–442, 1986.

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